Mem. S.A.It. Vol. 87, 579

Memorie della

Internal dynamics of globular clusters

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Abstract. This background paper summarises the variety of tools with which the dynamical evolution of globular star clusters can be studied, ranging from *N*-body techniques to fast "synthetic" methods. We also review the various dynamical processes at work, including not only two-body relaxation, but also the gravothermodynamics which underpins our understanding of these complicated models. These concepts are illustrated by application to the evolution of recent high-resolution simulations and their stellar-mass black holes.

Key words. Galaxy: globular clusters: general – methods: numerical – stars: black holes

1. Introduction

The study of the stellar dynamics of rich star clusters is a mature subject, but remains a formidable computational challenge. Nevertheless, the results of such computations are understandable on the basis of a small number of principles. In this paper we attempt to summarise these principles and apply them to some results of simulations, with some emphasis on the stellar-mass black holes which many globular clusters are now expected to retain. In a sense we aim to update a paper of King (2008), who showed the simple dynamical principles underlying the sequence of King models (King 1966), but now we consider the evolutionary aspects of the dynamics instead of focusing on equilibrium models.

2. Methods

Table 1 lists most of the methods that are currently available (but not always in the public domain) for modelling the dynamical evolution of rich star clusters. In the last column are given examples of specific clusters for which tailored simulations have been computed. Naturally, these methods have also been used for numerous other studies on the evolution of globular clusters in general.

While *N*-body techniques might seem the method of choice, they are still very slow, except for relatively small objects with long evolutionary time scales (such as the two Palomar clusters mentioned). It must be borne in mind that the search for suitable initial conditions requires many trials, even though these may be guided by faster, simpler methods, and so methods such as Monte Carlo will not be replaced by *N*-body techniques in the near future. Indeed the two large *N*-body simulations mentioned in the last column of the Table used initial conditions derived from numerous Monte Carlo runs. It is tempting to cut corners by scaling from smaller *N*-body models, but the various processes which drive dynamical evolution (Sec.3) do not scale with *N* in the same way. When scaled models are adopted,

Table 1. Methods for simulating dynamical evolution of star clusters

Technique	Pros	Cons	Examples of specific clusters
N-body	Gold standard Freely available	Takes months/years Scaling tricky	Pal 14, Pal 4 (Hasani Zonoozi et al. 2011, 2014), M4 (Heggie 2014, 2015), NGC 4372 (Wang et al. 2016)
Monte Carlo	Takes day(s) Freely available (MOCCA)	No rotation Tides tricky Needs N-body checks	M4 (Heggie & Giersz 2008) NGC 6397, 47 Tuc (Giersz & Heggie 2009, 2011) M22 (Heggie & Giersz 2014)
Fokker-Planck	Simple No noise	Slowed by binaries, MF	M71 (Drukier, Fahlman, & Richer 1992) NGC 6624 (Grabhorn et al. 1992) M15 (Grabhorn et al. 1992; Phinney 1993 Dull et al. 1997; Murphy, Cohn, & Lugger 2011)
Gas	Even simpler No noise	Slowed by binaries, MF	M3 (Angeletti, Dolcetta, & Giannone 1980)
Synthetic	Takes msec Freely available (EMACSS)	Global values only	M4, 47 Tuc, NGC 6397, M22, ω Cen, Pal 14, Pal 4, G1 And (Pijloo et al. 2015)

it seems essential at least to mention this difficulty.

The Monte Carlo technique is surely the method of choice at present. Because of the assumptions and approximations which it invokes, some problems (such as rotation) are quite beyond reach, but also it seems prudent to subject it to ongoing checks within the range of N where it can be compared with "identical" N-body models. While this has been done in several studies (Heggie et al. 1998; Giersz, Heggie, & Hurley 2008; Giersz et al. 2013; Rodriguez et al. 2016), such comparisons are not unlimited, and may not include outputs which are of importance in some new investigation.

While the lists of specific cluster simulations in the last column of Table 1 are thought to be complete for most of the methods, this is not true for the Fokker-Planck model. Examples of Fokker-Planck simulations on other clusters can be found by searching the publications of the lead authors noted in that entry of the Table.

3. Applying the principles of collisional stellar dynamics

Computations supply much data but limited understanding. Our purpose in this section is to study one particular large N-body simulation and show how essential aspects of its evolution can be understood on the basis of simple principles. Accounts of these principles can be found in Spitzer (1987), Binney & Tremaine (2008) and Heggie & Hut (2003), as well as in innumerable original papers.

The simulation (Heggie 2014) was intended to provide a model for the Galactic globular cluster M4, though at the age of M4 its central surface brightness turned out to be too high. The initial conditions for this simulation, which had been suggested by a Monte Carlo study (Heggie & Giersz 2008), were compact, with initial half-mass relaxation time $t_{rh}(0) \simeq 0.12$Gyr. We also remark on evolutionary models of two other, more slowly evolving clusters: a Monte Carlo model of 47 Tuc (Giersz & Heggie 2011, $t_{rh}(0) \simeq 0.7$Gyr) and an N-

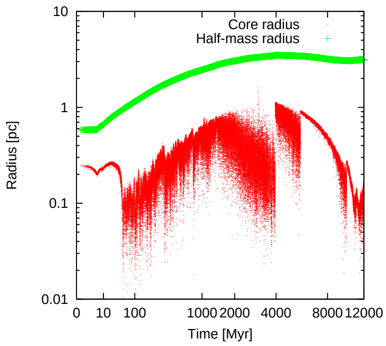

Fig. 1. Evolution of the core and half-mass radii for an N-body model of the evolution of M4. The features in the core radius at about 4, 5 and 10 Gyr are spurious (see Heggie 2014). Other features are real, and discussed in the text.

body model for NGC 4372 (Wang et al. 2016, $t_{rh}(0) \simeq 7$Gyr).

The evolution of the core and half-mass radii is shown in Fig.1. Up to about 5 Myr the core radius decreases because of the segregation of massive stars. This is a consequence of two-body relaxation in a multi-mass system, and takes place on a time scale of a fraction of $t_{rh}(0)$. Thereafter, internal evolution of the massive stars leads to heavy loss of mass from the cluster. This injects energy at a rate

$$\dot{E} \simeq \phi_c \dot{M}, \qquad (1)$$

where ϕ_c is the potential at the point where the mass is lost. Note that ϕ_c and $\dot{M}$ are negative, and $\dot{E} > 0$. This heating gives rise to an almost homologous expansion of both the half-mass and core radii.

Two-body relaxation implies an outward flow of heat, on the relaxation time scale t_{rh}, i.e. it is of order $|E|/t_{rh}$. This is a constant requirement of an evolving cluster, and evidently it cannot be met indefinitely by the mass loss through stellar evolution, which quickly diminishes. Therefore the core collapses, as one sees from the evolution of its radius. In this phase its membership becomes dominated by the stellar-mass black hole remnants of massive stars. Though the total mass of the core diminishes rapidly, so does its radius, and the rate of three-body interactions rises sufficiently for the creation of black-hole binaries. Both this process, and subsequent interactions with other stars/remnants in the core, are exothermic, and bring core collapse to a halt at "core bounce", which occurs in this model at about 50 Myr. Not only are these processes exothermic; they also cause the ejection of binaries and single stars, and the result is that eq.(1) still applies, albeit with a small additional numerical factor on the right, and now ϕ_c refers to the potential in the core.

Hénon (1975) left us a powerful insight (which the author refers to as "Hénon's Principle") that the subsequent evolution of the core would (or at least could) be self-regulating. A dense core produces too much energy for the required outward flow of heat, and causes core expansion, which moderates the energy production. An underdense core, on the other hand, collapses again. Thus, when the core behaves in this way (referred to as "balanced evolution"), it constantly adjusts to produce the flux of energy required by relaxation. It follows from the above arguments that the rate of mass loss is given, in order of magnitude, by equating $\phi_c \dot{M}$ and $|E|/t_{rh}$, i.e. $\dot{M} \simeq E/(\phi_c t_{rh})$. In the present context this mass loss consists of escaping black holes, and so their rate of loss is largely governed by bulk properties of the cluster; the only dependence on details of the core is the weak dependence on the central potential.

Balanced evolution is subject to gravothermal instability (Sugimoto & Bettwieser 1983), and may only be balanced on average. Indeed the core radius in the M4 model is subject to large quasi-periodic excursions (gravothermal oscillations) from the moment of core collapse for at least 1 Gyr (Fig.1).

The loss of black holes implies that their ability to sustain the flow of energy by relaxation is increasingly stretched. By about 4 Gyr the expansion of the core radius is reversed. Now Hénon's observations on the self-regulating nature of core evolution require its recollapse, until some other process of energy generation becomes effective enough. Eventually (around 10 Gyr) this is provided by the small percentage of primordial binaries present in this model. For the last 2 Gyr or so of its life, they restore balanced evolution, though again it is subject to oscillatory instability. At this point only a few stellar-mass black holes remain.

What has been said about this model of M4 applies qualitatively in the same way to the evolution of the quoted models of 47 Tuc and NGC 4372. The main difference stems from the overall longer time scales of these two clusters. 47 Tuc, for example, has just about reached the point where the evolution of the core radius begins to turn into a contraction, but the Monte Carlo model predicts that the collapse will take many more Gyr. In NGC 4372, on the other hand, the collapse of the system of stellar-mass black holes takes about 1 Gyr (as opposed to about 50 Myr in M4), and even now the evolution of the cluster is mainly sustained by its stellar-mass black holes.

4. Conclusions

In the modelling of the dynamical evolution of rich star clusters, N-body techniques represent the gold standard, but for rich systems they are greatly outpaced by Monte Carlo methods. Both techniques have been applied to a wide variety of problems, but so far the number of models tailored to specific clusters is limited (Table 1).

Evolutionary models produce vast amounts of information relevant to a wide variety of problems, but the basic principles on which a cluster evolves are relatively straightforward. Even the evolution of the number of stellar-mass black holes follows simple ideas.

Acknowledgements. I thank R. Capuzzo-Dolcetta for an interesting comment about the cited literature, and A.L. Varri for linguistic advice.

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